Complementarities of different types of capital in public sectors

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Motivation

- bulk of the literature analyzes the effect of intangible investment and intangible capital for economic growth (Roth and Thum 2013, Görzig and Gornig 2013, Corrado et al. 2013, Edquist 2011, Corrado et al. 2009, Marrano et al. 2009,...)
- substitutability between intangible and tangible capital is not studied
- knowing the elasticity of substitution between these two inputs is essential ...

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- knowing the elasticity of substitution between these two inputs is essential ...

  ... especially in the public sector, otherwise stimulus packages or spending cuts could have unintended consequences

Research question

- What is the elasticity of substitution between intangible and tangible capital?
  or in other words:
  Are tangible capital and intangible capital substitutes or complements?

Substitution Measures

Elasticity of substitution

- shows "the ease with which the varying factor can be substituted for others" (Hicks, 1932: p.117), or,
- it "measures the degree to which the substitutability of one factor for another varies as the proportion between the factors varies" Lerner (1933, 68), or, in other words,
- it measures the percentage change in factor proportions due to a change in marginal rate of technical substitution, or,
- it is effectively a measure of the curvature of an isoquant (Lerner, 1933).

In essence there are three measures of the elasticity

- *Direct* elasticity of substitution (DES),
- *Allen* elasticity of substitution (AES)
- *Morishima* elasticity of substitution (MES)

Elasticities of Substitution

Direct elasticity of substitution (DES) : $$\sigma_{ij}^{D} = \frac{f_i X_i + f_j X_j}{X_i X_j} \frac{F_{ij}}{F} \quad (1)$$

Allen elasticity of substitution (AES) : $$\sigma_{ij}^{A} = \frac{\sum_k^n f_k X_k}{X_i X_j} \frac{F_{ij}}{F} \quad (2)$$

Morishima elasticity of substitution (MES) : $$\sigma_{ij}^{M} = \frac{f_j}{X_i} \frac{F_{ij}}{F} - \frac{f_j}{X_j} \frac{F_{jj}}{F} \quad (3)$$

with f_i is the partial derivative of the production function f with respect to input i, F is the determinant of the bordered Hessian matrix H and F_{ij} is the cofactor of H, X_i is input i

- in the two input case, AES corresponds to DES $\left(\sigma_{ij}^{A} = \sigma_{ij}^{D}\right)$.
- DES and AES are symmetric, MES is not symmetric $\left(\sigma_{ij}^{A} = \sigma_{ji}^{A} \text{ and } \sigma_{ij}^{D} = \sigma_{ji}^{D}, \sigma_{ij}^{M} \neq \sigma_{ji}^{M}\right)$.

Hessian Matrix

Production Function

- CES production functions not suitable for the analysis as they assume constant elasticity of substitution (e.g. CD assumes ES of one).
- translog production function is sufficiently flexible (Christensen et al. 1971, 1973).

$$y = \alpha_0 + \sum_{i=1}^{n} \alpha_i x_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} x_i x_j, \tag{4}$$

- estimation using structural approach along the lines of Olley and Pakes (1996) and Ackerberg, Caves and Frazer (2006)

Strcutural Approach Derivatives

Database Requirements

- real output
- labour in heads or working hours
- deflated stocks for tangible and intangible capital plus deflated investments in both capitals stocks
- all variables in common currency for all countries and sectors
- data for the entire public sector or at one-digit sector level

Testing the Empirical Strategy

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Practical Steps

1. estimate $f()$ as Cobb-Douglas (CD) and Translog (TL)
2. testing whether CD or TL applies
3. estimating elasticities (E), marginal product (MP), marginal rate of technical substitution ($MRTS$) etc., calculate first and second derivative
4. construct bordered Hessian matrices
5. calculate DES, AES and MES
6. asses the elasticity of substitution between inputs

Testing the Empirical Strategy

- Cobb-Douglas production function:

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_c c_{t,jt} + \beta_i c_{i,jt} + \omega_{jt} + \varepsilon_{jt} \tag{5}$$

- Translog production function:

$$y_{jt} = \beta_0 + \beta_l l_{jt} + \beta_c c_{t,jt} + \beta_i c_{i,jt} + \frac{1}{2}\beta_{ll} l_{jt}^2 + \frac{1}{2}\beta_{cc} c_{t,jt}^2 + \frac{1}{2}\beta_{ii} c_{i,jt}^2 + \beta_{lc} l_{jt} c_{t,jt} + \beta_{li} l_{jt} c_{i,jt} + \beta_{ci} c_{t,jt} c_{i,jt} + \omega_{jt} + \varepsilon_{jt}, \tag{6}$$

with y_{jt} for value added, l_{jt} as labour input, $c_{t,jt}$ as tangible capital input, $c_{i,jt}$ as intangible capital, with ω_{jt} as productivity, ε_{jt} as iid error component, j and t as country and time index, respectively.

Table 1: Cobb-Douglas and Translog production function (OLS)

	CD		Translog	
variables	coeff.	Std. Err.	coeff.	Std. Err.
Intercept	0.70397***	0.13994	-2.88747	3.42519
$Capital_T$	0.39533***	0.03402	2.77645*	1.56196
$Capital_I$	0.35415***	0.02012	-1.11082*	0.64570
Labour	0.28453***	0.01732	-0.57432	0.79340
$0.5 \times Capital_T^2$			-1.04369***	0.38129
$0.5 \times Labour^2$			-0.38691***	0.06849
$0.5 \times Capital_I^2$			-0.57960***	0.16870
$Capital_T \times Labour$			0.45566***	0.13780
$Capital_T \times Capital_I$			0.62393***	0.23131
$Capital_I \times Labour$			-0.08220	0.08410

*** $p < 0.01$, ** $p < 0.05$, ** $p < 0.1$

Testing the Empirical Strategy

- testing whether CD or Translog applies by means of *Wald − test* and *Likelihood ratio test*
  - *likelihood ratio test* rejects Cobb-Douglas model with $\chi^2 - value$ of 79.198 and a $p - value$ of < 0.001
  - *Wald − test* also rejects Cobb-Douglas model with an $F - value$ of 17.098 (6) and a $p - value$ of < 0.001

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- tests confirm to use Translog and to proceed with estimation strategy

Testing the Empirical Strategy

Table 2: Initial key figures

Key figures	$Capital_T$	$Capital_I$	$Labour$
Aver. Output elasticities	0.289	0.372	0.352
Aver. Marginal products	0.173	11.137	1.519

- if intangible capital increase by 1 percent, the output of the business sector will increase by 0.37 percent on average
- if tangible capital input increase by one unit, the output will increase by 11 units on average

Elasticities Figure

Testing the Empirical Strategy

Table 3: Average and median for AES and MES

AES	Median		Mean	
	$Capital_I$	*Labour*	$Capital_I$	*Labour*
$Capital_T$	-0.0213	0.2865	0.0207	0.1723
Labour	0.5806	-	0.5672	-
MES	Median		Mean	
	$Capital_I$	*Labour*	$Capital_I$	*Labour*
$Capital_T$	0.3334	0.2832	0.3305	0.3195
Labour	0.5082	-	0.5213	-

- tangible capital and labour weak are substitutes for each other
- labour and intangible capital are moderate substitutes for each other
- difference between AES und MES with respect to elasticity of substitution for both capital types

Summarizing

Interpretation

- when the private sector invest in tangible capital (e.g. supported by subsidies) it should also invest in intangible capital in order to use additional tangible capital efficiently
- assuming similar results for the public sector: efficient use of additional input (e.g expansionary fiscal policy) only if additional spendings also for labor and intangible capital

Methodological conclusion

- method & code works in general
- potentially problem regarding significance of coefficients when estimating translog production functions
- caution in deriving policy recommendation, because results might differ depending on the applied measure of elasticity

Currents Status and Next Steps

Current status

- literature review almost complete
- econometric approach developed
- approach largely coded
- econometric and method sections partly written

Next steps

- finalize coding
- start analysis for public sector once data are available
- writing of SPINTAN discussion paper on ES between tangible and intangible capital in the public and private sector

Thank you for your attention!

Hessian Matrix

The bordered Hessian matrix is defined as follows:

$$H = \begin{bmatrix} 0 & f_1 & f_2 & \dots & f_N \\ f_1 & f_{11} & f_{12} & \dots & f_{1N} \\ f_2 & f_{12} & f_{22} & \dots & f_{2N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_N & f_{N1} & f_{N2} & \dots & f_{NN} \end{bmatrix} \tag{7}$$

with f_i is the partial derivative of f with respect to input i and f_{ij} is the partial derivative of f_i with respect to the jth input.

Back to ES

Hessian Matrix

The cofactor F_{ij} for a Hessian matrix is derived as

$$F_{ij} = (-1)^{i+j} \cdot \begin{vmatrix} 0 & f_1 & \dots & f_{j-1} & f_{j+1} & \dots & f_N \\ f_1 & f_{11} & \dots & f_{1,j-1} & f_{1,j+1} & \dots & f_{1N} \\ f_2 & f_{12} & \dots & f_{2,j-1} & f_{2,j+1} & \dots & f_{2N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{i-1} & f_{1,i-1} & \dots & f_{i-1,j-1} & f_{i-1,j+1} & \dots & f_{i-1,N} \\ f_{i+1} & f_{1,i+1} & \dots & f_{i+1,j-1} & f_{i+1,j+1} & \dots & f_{i+1,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ f_N & f_{N1} & \dots & f_{N,j-1} & f_{N,j+1} & \dots & f_{NN} \end{vmatrix} \tag{8}$$

Back to ES

Structural Estimation Approach

- structural approach along the lines of OP (1996) and ACF (2006)
- function of observable used in 1st step in order to control for unobserved productivity, thus overcoming simultaneity and endogeneity problem because ω_{jt} is omitted

$$i_{jt} = f_t(\omega_{jt}, c_{t,jt}, c_{i,jt}, l_{jt}); \text{ inverted: } \omega_{jt} = f_t^{-1}() \quad (9)$$

$$y_{jt} = \beta_l l_{jt} + ... + f_t^{-1}() = \phi_t(i_{jt}, c_{t,jt}, c_{i,jt}, l_{jt}) + \varepsilon_{jt} \quad (10)$$

with $\phi_t() = \beta_l l_{jt} + \beta_c c_{t,jt} + \beta_i c_{i,jt} + ... + f_t^{-1}(i_{jt}, c_{t,jt}, c_{i,jt}, l_{jt})$ and ε_{jt} iid error term

Back to Poduction Function

Structural Estimation Approach

- second stage assumes first-order Markov process for ω_{it} (OP, 1996)
- expectation about productivity depends on past productivity and "innovation":

$$\omega_{jt} = E(\omega_{jt} | I_{jt-1} + \xi_{jt}) \tag{11}$$

approximated by AR(1) process:

$$\omega_{jt} = g_t(\omega_{jt-1} + \xi_{jt}) \tag{12}$$

- g_t approximated non-parametrically by (PPL, 2004):

$$\omega_{jt} = \lambda_0 + \lambda_1 \omega_{jt-1} + \lambda_2 \omega_{jt-1}^2 + ... + \epsilon_{jt} \tag{13}$$

- it follows from Eq. (9) and Eq. (10) that ω_{jt} can be substituted by $\phi_{it} - \beta_l l_{jt} - \beta_c c_{t,jt} - \beta_i c_{i,jt} - ... - \beta_{ci} c_{t,jt} c_{i,jt}$
- Eq. (13) is estimated by means of GMM

Back to Poduction Function

Derivatives

The first derivative of translog production function with respect to the ith input is

$$f_i = MP_i = \varepsilon_i AP_i = \left(\alpha_i + \sum_j^n \alpha_{ij} x_{ij} \right) \frac{Y}{X_i}. \tag{14}$$

The second derivatives of a translog production function with n inputs are

$$f_{ij} = \frac{\partial^2_Y}{\partial X_i \partial X_j} = \frac{Y}{X_i X_j} \left(\alpha_{ij} + \epsilon_i \epsilon_j - \delta_{ij} \epsilon_i \right) \tag{15}$$

or

$$f_{ij} = \frac{\alpha_{ij} Y}{X_i X_j} + \frac{MP_i MP_j}{Y} - \delta_{ij} \frac{MP_i}{X_i}, \tag{16}$$

where δ_{ij} is the Kronecker's delta with

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} . \tag{17}$$

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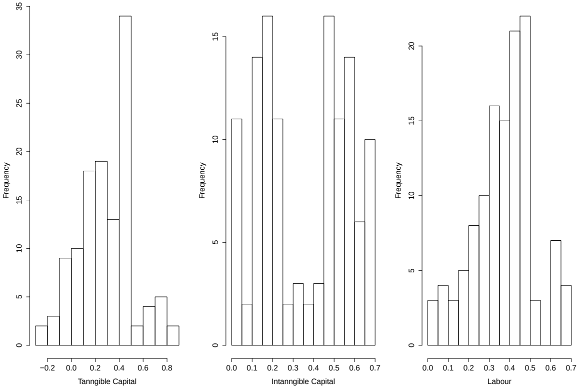
Testing the Empirical Strategy

Figure 1: Output elasticites

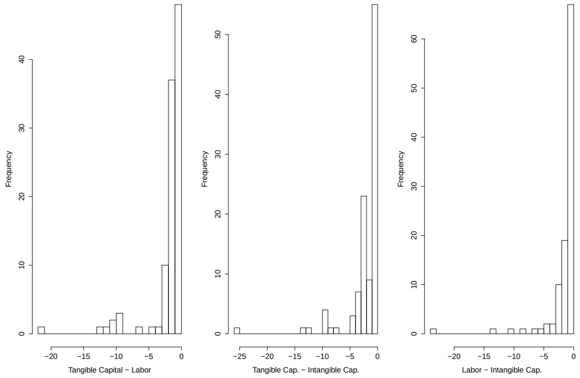


- monotonicity condition not fulfilled for 14 observations

Back to output elasticity

Testing the Empirical Strategy

Figure 2: Relative Marginal Rate of Technical Substitution



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Testing the Empirical Strategy

Table 4: Relative Marginal Rates of Technical Substitution (RMRTS)

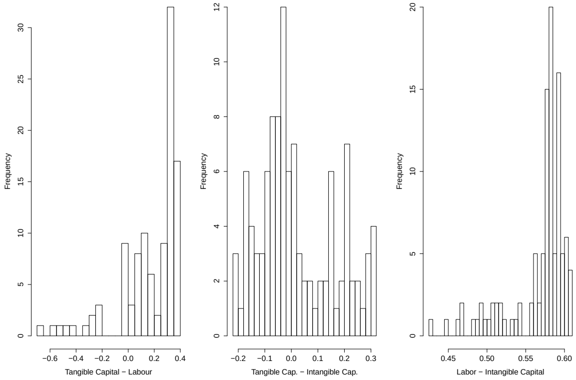
	$Capital_T$	$Capital_I$	$Labour$
$Capital_T$	-	1.4937	0.9585
$Capital_I$	0.6695	-	1.5107
$Labour$	1.0433	0.6620	-

- the reduction of labour input by one percent, requires to use - on average - around 0.66 percent more capital in order to produce the same amount of output as before

Back to ES

Testing the Empirical Strategy

Figure 3: Allen Elasticities of Substitution



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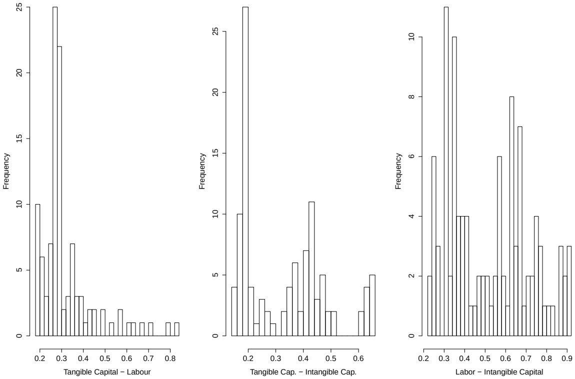
Testing the Empirical Strategy

Figure 4: Morishima Elasticities of Substitution



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