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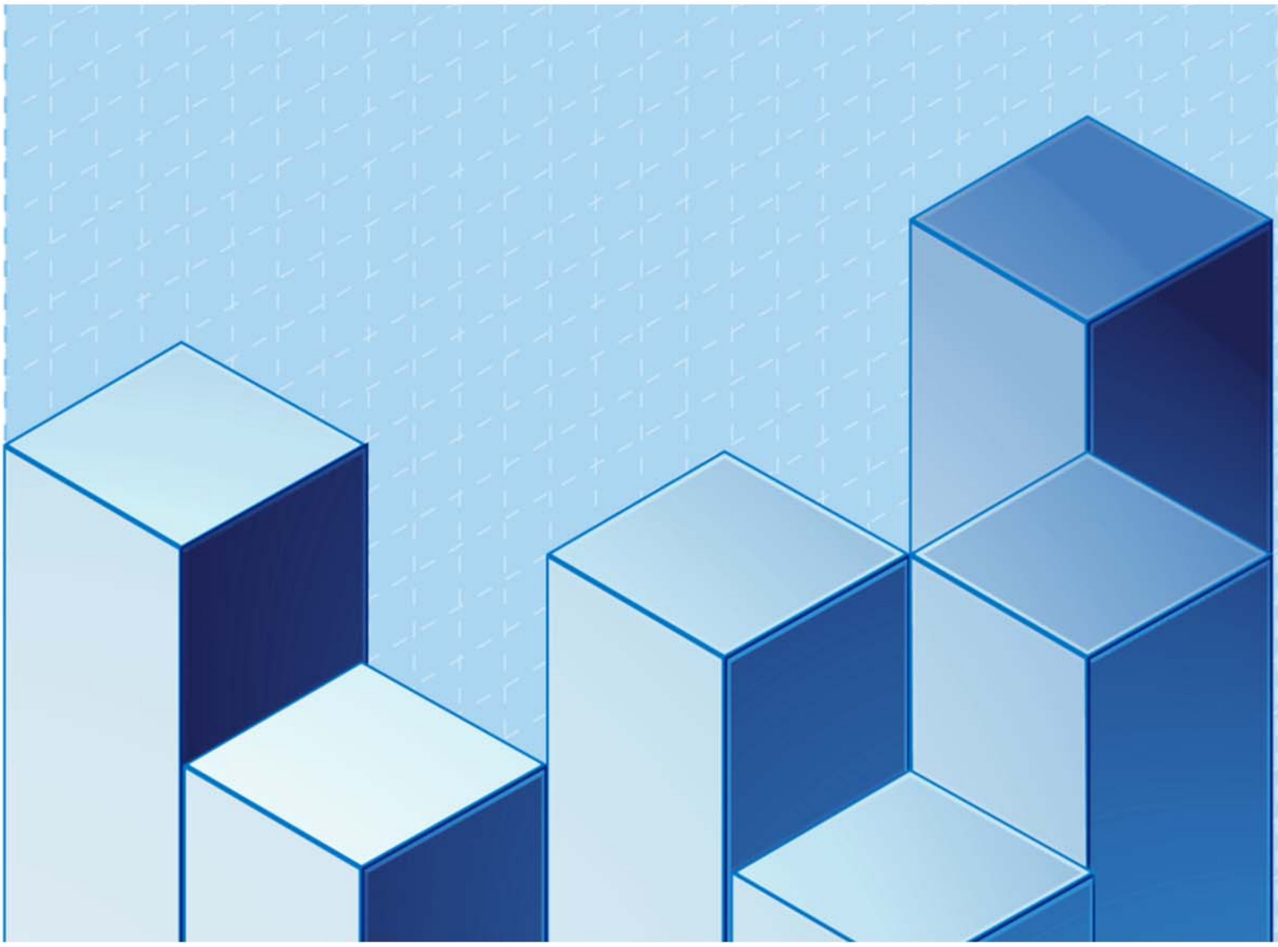

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THE EFFECTS OF SURVIVAL RATES ON EDUCATION IN A SIMPLE LIFE-CYCLE MODEL*

Martin Weale†

Abstract

This paper explores the decision to invest in education and its link with life expectancy and mortality. It presents evidence for a statistical relationship between life expectancy and participation in education in the European Union, and explores this in a life-cycle model. It is found that conventional estimates of returns to education imply much longer periods of education than are observed in practice. More conventional results can be found if the marginal return to education is assumed to decline with the age at which it is completed. Even then declining mortality accounts only for a small part in the extension of education seen since the early twentieth century. If, as has been suggested by some research, education itself influences mortality, then it is possible to account for over 60% of the increase in years of education as higher productivity and living standards have made life more fully worth living.

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† National Institute of Economic and Social Research

1 Introduction

It has been accepted for fifty years that there is an important investment aspect to participation in education. People build up their human capital by being educated and, as a result they expect to earn more subsequently. It was similarly established (Mincer 1974) that education does result in subsequent financial advantage, or at least that people who have studied for longer appear to earn more. There are questions of reverse causation and the suggestion that education is no more than a filtering device (Spence 1973). More recent work, however, which uses matching methods to compare people who have similar characteristics generally but who differ in terms of education confirms the idea that education leads to higher earnings, as do studies which use instrumental variables (Dickson 2013).

These studies of returns to education do not explain the extent to which people avail themselves of educational opportunities. An early study (Ben-Porath 1967) suggested that rising life expectancy was likely to be a factor behind increasing participation in education and increasing average duration of education. The logic was that as survival rates rose, the expected period over which the financial returns to education could be enjoyed would increase. This would make participation in education more worthwhile, and increase the average duration of education.

Hazan (2009) argued that, while the logic of this argument was sound, there was a flaw. Even while life expectation had risen sharply, the expected number of hours worked during the lifetime had typically declined. Without some additional explanation, such as the idea that education also increased the return to leisure, it was hard to see that an increasing life-span would lead to rising education if the number of paid hours of work had actually declined. More recently Cervellati & Sunde (2013) argued that the logic behind this argument was itself flawed. What mattered was the balance between the marginal benefit of extra education and the marginal cost in terms of earnings foregone. It was perfectly possible for this marginal condition to point to increasing education even when the expected number of hours worked over the lifetime was falling rather than rising.

This discussion naturally raises the idea that decisions about participation in education should be examined in a context where retirement is endogenous rather than exogenous. Bloom et al. (2014) explore the relationship between retirement, health and mortality patterns. In this paper I take their model, extending it to show the education decision as well as the retirement decision. The extended model shows the decision to invest in education modelled in exactly the way that Cervellati & Sunde (2013) suggest. The model is calibrated to fit data for the United Kingdom and the effects of rising life expectancy are investigated.

There is evidence to suggest that education has a direct impact on life expectancy. UK mortality statistics show a clear link between social class and life expectancy in the United Kingdom; thus between 1997 and 2001 a professional man had a life expectancy five years higher than that of a manual worker. Using the British Household Expenditure Survey Dorsett et al. (2014) found that men with higher education qualifications in the United Kingdom had a life expectancy, at age sixty-five, of over four years longer than that of a man with only minimal qualifications. Lleras-Muney (2005) suggests that one year of education raises life expectancy at the age of thirty-five by up to 1.7 years, based on an assessment of the effects of different compulsory education laws in different states in the United States. On the other hand it cannot be said that the evidence is completely unambiguous. Clark & Royer (2013) studied the impact of increased years of education as a result of the changes to the school-leaving age in 1947 and 1972 in the United Kingdom. Comparing the mortality patterns of the cohorts affected by the change with those too old to be affected, they find that, if anything, the increase in compulsory education in 1947 had the effect of increasing mortality rates between the ages of forty-five and sixty-nine slightly. This, however, relates to an age range when mortality rates are in general low; their data did not allow them to look at the impact on mortality rates of men in their seventies and eighties.

The model we develop can be extended to examine the effects of any link between education and life expectancy, on the assumption that people choose their education with reference to the extra life that it brings as well as with reference to the enhanced earnings that results from it. Simulation results from the model, structured round mortality rates for men in the United Kingdom, are compared with those found from an empirical analysis of educational investment and life expectancy for pooled data from the European Union.

Section 2 of this paper presents some evidence on the statistical relationship between educational participation and life expectancy. Section 3 sets out the theoretical model. Section 4 examines some of the implications of relationship between returns to education and the rate of return on capital, and suggests that credible results can be obtained only if returns to education are declining years of education. Section 5 discusses the modelling of mortality, while Section 6 presents simulations exploring the relationship between health, survival, retirement and educational participation, and Section 7 concludes.

2 Evidence from the European Union

Rising educational attainment has run in step with reduced mortality in most advanced economies for quite a long period, and it is not therefore straightforward to identify causative effects from the latter to the former. In this section we simply attempt to show a relationship. We then compare the magnitude of the effect what emerges from simulations of our theoretical model, in order to understand whether reduced mortality can be seen as a plausible factor contributing materially to the rising participation in non-compulsory education.

Taking data from the Eurostat web site we compare, in figure 1 the change in the log of the odds ratio of the proportion of men aged fifteen to twenty-four participating in education between 2005 and 2012 with the log change in life expectancy at fifty over the same period; we focus on men because the subsequent analysis looks at retirment in a simple model which does not take account of the uneven employment pattern many women have experienced historically. Of course, for there to be some causal link, it would be necessary to assume that people making educational decisions were drawing inferences about their own survival prospects from the forecast life expectancy of men aged fifty. As it is, with data for nineteen EU countries[1] we find evidence of a statistically significant relationship ($t_{17} = 2.24$, $P = 3.9\%$).

[1] Belgium, Czech Republic, Denmark, Germany, Eire, Greece, Spain, France Italy, Hungary, Netherlands, Austria, Poland Portugal, Slovenia, Slovakia, Finland, Sweden, United Kingdom

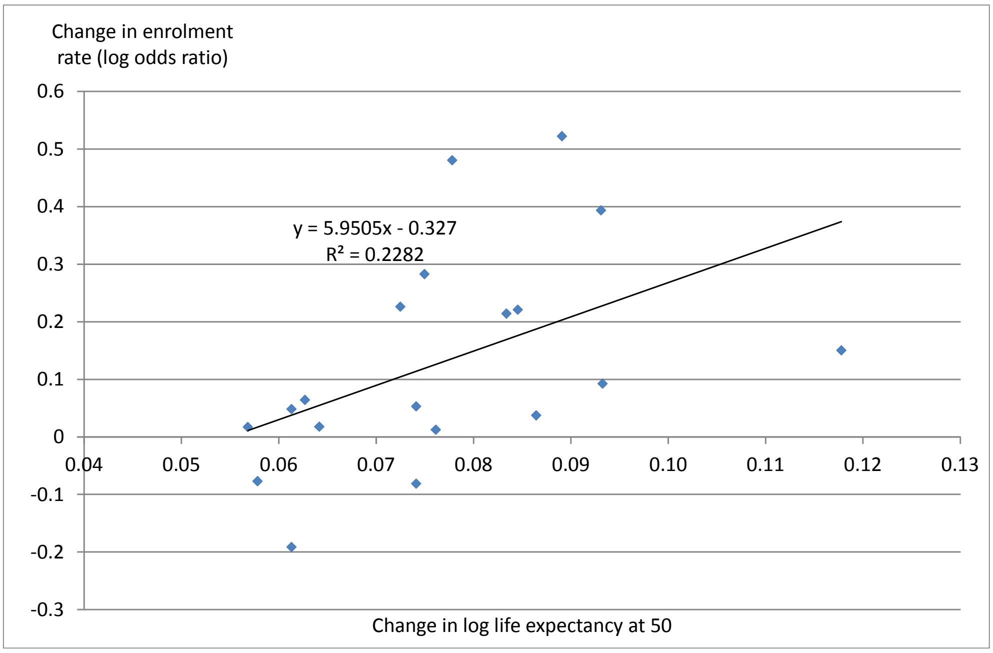

Figure 1: Education Participation and Men's Life Expectancy in the EU; Changes from 2005 to 2012

3 The Theoretical Model

This section presents a life-cycle model which makes it possible to explore the interplay between survival prospects- i.e. mortality rates, retirement and educational decisions. It is a development of the model described by Bloom et al. (2014).

The model explores the decisions of a single consumer. Each consumer is assumed to derive utility from consumption and disutility from work. The utility function is assumed to be separable between consumption, c_t, and work. Utility from consumption is assumed to be modelled with a constant elasticity of substitution, while that from work is taken to be the product of the proportion of the year spent in education, ζ_t or working, χ_t and a function $\phi(h_t)$, of health state h_t.

Bloom et al. (2014) suggest a utility function in which utility takes a constant elasticity of substitution specification in consumption, but is linear in the proportion of total time not available for leisure, i.e. devoted to study or work.

$$U = \frac{c_t^{1-\sigma}}{1-\sigma} - (\zeta_t + \chi_t)\,\phi(h_t)$$

In what follows we set out the model with σ >1; in the limiting case of $\sigma = 1$, $\frac{c_t^{1-\sigma}}{1-\sigma}$ is replaced by $\log c_t$.

If the health state is defined so that a higher health state represents better health, then we expect that

$$\frac{d\phi(h_t)}{dh_t} < 0$$

The disutility of work is lower for people in good health than for those in bad health while the opposite is true of the wage rate.

This specification implies that, if $\sigma > 1$, utility is always negative, with the implication that an increase in life-span would make total life-time utility lower rather than higher. The function is not suitable for analysing marginal decisions which affect actual or expected life-span as we do when we discuss the possible endogeneity of mortality rates. Negative utility means that, far from any increase in survival being valued, it becomes something to be avoided. Murphy & Topel (2006) proposed a solution to this problem. They introduced a positive term, $\frac{\tilde{c}^{1-\sigma}}{1-\sigma}$, making the utility function

$$U = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\tilde{c}^{1-\sigma}}{1-\sigma} - (\zeta_t + \chi_t)\,\phi(h_t)$$

This implies that, when $\zeta_t = \chi_t = 0$ so that time is enjoyed rather than being devoted to either work or study, then utility is positive provided that $c_t > \tilde{c}$. The level of consumption $\tilde{c}$ carries a natural interpretation as representing a subsistence level of consumption at which life becomes not worth living and the lower this is, the higher is the level of utility associated with any actual level of consumption. In turn this means that, the lower is the value put on subsistence consumption relative to actual consumption, the greater is the value put on an extra year of life. The value of $\tilde{c}$ can be selected, along with the other parameters, with reference to the capacity of the model to fit the data.

In explaining the model we make the assumption that life is sequential, so that retirement follows work which follows education, and that in each phase of life time is fully allocated in that way. It is possible to show that the optimal conditions lead to time being fully allocated in one of these three activities, but that adds unnecessary complexity. We do discuss subsequently, a condition which has to be met if people are not going to start working again late in life.

The wage rate is w_t. This is a function of past education. The influence of education on wages is, as with most models of returns to the duration of education, assumed to be proportionate. Thus the partial influence of education on earnings is, with $\theta(\tau)$ the Mincerian rate of return on education with τ years of education,

$$\frac{\partial w_t}{w_t} = \theta(\tau);\ t \geq \tau.$$

This allows for the possibility of a non-linear relationship, with the non-linearity being a function of time rather than past cumulated education. The importance of this is explored subsequently.

The wage rate is also assumed to rise over time as a result of the influence of a multiplicative factor, e^{gt} where g is the rate of exogenous labour-saving technical progress. Thus

$$w_t = e^{gt} w_0 e^{\int_0^{t_1} \theta(\tau) d\tau}.$$

3.1 The Basic Model

We first consider a situation where survival is independent of education. The exogenous probability of surviving to age t is s_t and the discount rate is δ. There is a maximum possible age, T beyond which $s_t = 0$. It is assumed that the real rate of interest is r, but that, following Yaari (1965) people can invest in fair annuities, so that the savings of each cohort are pooled and, as members of the cohort die, the savings of the dead members of the cohort are allocated to the survivors.

This means that the consumer faces the following optimisation problem

$$Max \int_0^T s_t e^{-\delta t} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\tilde{c}^{1-\sigma}}{1-\sigma} - (\zeta_t + \chi_t)\, \phi(h_t) \right\} dt$$

subject to the budget constraint

$$\int_0^T (e^{gt} \chi_t w_0 e^{\int_0^{t_1} \theta(\tau) d\tau} - c_t) s_t e^{-rt} dt \geq 0$$

We assume that except notionally at the crossover points when the individual changes from education to employment or from employment to retirement, he/she is either fully occupied with education ($\zeta_t = 1, \chi_t = 0$), employment ($\zeta_t = 0, \chi_t = 1$) or leisure ($\zeta_t = 0, \chi_t = 0$) . Further these three phases of life come in sequence and there are no reversals. Thus people do not move back into education after they have started employment, or return to work after they have retired. The first of these conditions is very intuitive. The way the model is specified, with no depreciation of human capital, the greatest benefits are obtained from education at the start of life. The second condition largely conforms to reality but the conditions needed for it to be true are discussed subsequently.

The Lagrangian problem is

$$Max \int_0^T s_t e^{-\delta t} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\tilde{c}^{1-\sigma}}{1-\sigma} - (\zeta_t + \chi_t)\, \phi(h_t) \right\} dt + \lambda \int_0^T (e^{gt} \chi_t w_0 e^{\int_0^{t_1} \theta(\tau) d\tau} - c_t) s_t e^{-rt} dt$$

Here λ is the marginal utility of wealth in period 0. Differentiating with respect to c_t

$$s_t e^{-\delta t} c_t^{-\sigma} = \lambda s_t e^{-rt} \tag{1}$$

We make the convenient assumption that $r = \delta$ which results in c_t being constant. We denote that as c^*.

The derivative with respect to t_1 is found by considering the effect of a marginal change in the time at which χ_t moves from 0 to 1, and comparing this with the impact on the value of discounted labour income over the remaining life-span. After taking account of the fact that $r = \delta$

$$s_{t_1} e^{(g-r)t_1} e^{\int_0^{t_1} \theta(\tau) d\tau} = \int_{t_1}^T e^{(g-r)t} \chi_t \theta(t_1) e^{\int_0^{t_1} \theta(\tau) d\tau} s_t dt. \tag{2}$$

The left-hand side of this expression says that the disutility of unit time devoted to education at age, t_1, just matches the marginal utility of the discounted extra earnings which result from the increment to time spent in education. As Cervellati & Sunde (2013) note, this condition does not necessarily imply that t_1 will increase only if subsequent working life increases.

Finally, the derivative with respect to t_2, the retirement age is found by considering the effect of a marginal extension of the time at which χ_t changes from 1 to 0. Once again, $r = \delta$

$$s_t \phi(h_t) = \lambda s_t e^{gt} w_0 e^{\int_0^{t_1} \theta(\tau) d\tau} \tag{3}$$

It is possible that technical progress may make re-entering work after a period of retirement attractive. That will be avoided if, for all $t > t_2$ we have

$$\phi(h_t) > \lambda e^{gt} w_0 e^{\int_0^{t_1} \theta(\tau) d\tau}.$$

There are then no further interior solutions. Similarly, for $t < t_2$ we require

$$\phi(h_t) < \lambda e^{gt} w_0 e^{\int_0^{t_1} \theta(\tau) d\tau}$$

to ensure that there is no interior solution before $t = t_2$. A sufficient condition for these requirements to be met is that the disutility of work increases rapidly enough with time, either because health status declines rapidly enough or because the effect of declining health status on the disutility of work is strong enough for that to be growing faster than does the benefit of work increase as a result of exogenous economic growth[2].

The first condition established that c_t is constant at c^* while the second and third allow us to determine the ages at which education ends, t_1 and retirement begins, t_2. To complete the picture we need to solve for the value of λ. With $\chi_t = 0$ if $t < t_1$ or $t > t_2$ and 1 otherwise, the budget constraint implies

$$\int_{t_1}^{t_2} (e^{gt} \chi_t w_0 e^{\int_0^{t_1} \theta(\tau) d\tau} - c^*) s_t e^{-rt} dt \geq 0$$

so that, at the optimum,

$$c^* = \frac{w_0 \int_{t_1}^{t_2} s_t e^{(g-r)t} \chi_t e^{\int_0^{t_1} \theta(\tau) d\tau} dt}{\int_0^T s_t e^{-rt} dt} \tag{4}$$

Equations (1), (2), (3) and (4) then enable us to solve iteratively for c^*, λ, t_1 and t_2 conditional on exogenous paths for the returns to education, health status and its effects, survival, and, of course the interest rate, growth rate and intertemporal elasticity of substitution. Not surprisingly, with mortality exogenous, none of these marginal conditions depends on subsistence consumption, $\tilde{c}$.

[2]It should be noted that if dw_0/dh_t is strictly positive rather than constant as assumed here, so that the fact that health declines with age means also that wages tend to decline with age, then this condition is more likely to be met.

If we now look at the influences on the retirement age, t_2, in equation (3), we can see that, conditional on the value of λ an increase in the initial wage, w_0 would result in a delay to retirement. However, from equation (4) we can see that with other things constant, an increase in w_0 raises c^* equiproportionately; we write $c^* = \kappa(t_1, t_2) w_0$. With the marginal utility of wealth declining in the level of consumption, that effect pulls in the opposite direction, reducing the marginal benefit of work at any age, and thus resulting in earlier retirement. We note that since $\lambda = u'(c^*) = u'(w_0\kappa(t_1, t_2))$ and thus (3) becomes

$$\phi(h_t) = u'(w_0\kappa(t_1, t_2))e^{gt}w_0 e^{\int_0^{t_1}\theta(\tau)d\tau}.$$

The effect on the right-hand side of a change in w_0 is

$$\begin{aligned}\frac{d\{u'(w_0\kappa(t_1, t_2))e^{gt}w_0 e^{\int_0^{t_1}\theta(\tau)d\tau}\}}{dw_0} &= u'(w_0\kappa(t_1, t_2))e^{gt}ew_0 e^{\int_0^{t_1}\theta(\tau)d\tau} \\ &+\kappa(t_1, t_2)\,u''(w_0\kappa(t_1, t_2))e^{gt}w_0 e w_0 e^{\int_0^{t_1}\theta(\tau)d\tau}\end{aligned}$$

If this is positive, then retirement is delayed (the value of $\phi(h_t)$ at which people retire is higher and they are older) while if it is negative then retirement is brought forward. For retirement to be delayed we require, noting that the second derivative of utility is negative

$$\frac{u'(w_0\kappa(t_1, t_2))}{w_0\kappa(t_1, t_2)\,u''(w_0\kappa(t_1, t_2))} < -1.$$

In other words, the intertemporal elasticity of substitution should be greater than 1. That is generally found not to be the case[3]. At the same time it should be noted that this result relies on the separability of the utility function between consumption and leisure.

Other things being equal, this result also applies to an extension of education as represented by an increase in t_1. Moreover, the retirement age, t_2 is measured in years from starting education (or from birth), not completing it. This means that, with an elasticity of substitution below one, an extension of education will result in a larger fall in working life than in the age of retirement.

From condition (2) we can see the issue behind the error in Hazan (2009) pointed out by Cervellati & Sunde (2013). An increase in future survival rates, s_t for $t > t_1$ must be balanced by a reduction in χ_t if the age at which education is completed is to be unchanged. The retirement age must, in other words fall. Looking at equation

[3] If the utility function is $u(w_0\kappa(\tau^*, t^*)) = \frac{(w_0\kappa(\tau^*, t^*))^{1-\sigma}}{1-\sigma}$, then this requirement is equivalent to $\sigma < 1$. Most estimates of σ are above 1.

(3) we can see that this requires λ the marginal utility of consumption to be lower, in other words for consumption to rise. That increase can in turn be achieved only if the age of completion of education does rise. The outcome of this process may well be earlier retirement and a longer period of education. Thus it is perfectly plausible for a longer period of education to be induced by rising survival rates even though there is some decline rather than an increase in the retirement age. Of course if survival rates also increase late in life, then the retirement wage will be increased.,The point is that improved life expectancy can induce a longer period of education even without an increase in the retirement age.

Before turning to the extension of the model to represent endogenous survival we discuss mortality functions. These become material to the marginal conditions once mortality rates are assumed affected by years of education.

3.2 Mortality Functions

3.2.1 Exogenous Survival

In order to implement the model it is necessary to adopt some functional form for survival. Bebbington et al. (2014) suggest a function in which at young ages the mortality rate is reasonably independent of age, as suggested by Yaari (1965) while in old age it follows more closely the function suggested by Gompertz (1825)

$$-\frac{d \log s_t}{dt} = m_t = \rho + \gamma e^{\kappa t}$$

This does not do a good job of fitting child mortality rates, but, to mortality data for a cohort aged sixteen and above, the three parameters can straightforwardly be fitted by non-linear least squares. Fitting the model in log terms ensures that $\rho > 0$ so that the model cannot generate negative mortality rates.

3.2.2 Endogenous Survival

The model can be adapted to address the situation in which survival rates are a function of educational attainment. Some relationship has to be proposed for the link between education and mortality.

We modify the earlier mortality function as follows. With an age t_1 on completion of education, the mortality rate is assumed to be defined by the expression

$$m_t = (\rho + \gamma e^{\kappa t}) e^{-\mu t_1}$$

One extra year of education depresses the log mortality rate by μ units. With very low mortality rates for people aged under sixty-five, however, it is likely that a proportional reduction in mortality rates, arising from an extension to education, will have the effect of tightening the budget constraint. People become only a little more likely to survive while of working age, but relatively more likely to survive for longer in retirement. Working in terms of survival rates rather than mortality rates, with $m_t = d\log s_t/dt$

$$\frac{d\log s_t}{dt} = -(\rho + \gamma e^{\kappa t})e^{-\mu t_1}$$

For $t > t_1$

$$\log s_t - \log s_{t_1} = -e^{-\mu t_1}\left[\rho(t-t_1) + \frac{\gamma}{\kappa}(e^{\kappa t} - e^{\kappa t_1})\right]$$
$$\frac{d\log s_t}{dt_1} - \frac{d\log s_{t_1}}{dt_1} = \rho e^{-\mu t_1} + \gamma e^{\kappa t_1}e^{-\mu t_1} + \mu e^{-\mu t_1}\left[\rho(t-t_1) + \frac{\gamma}{\kappa}(e^{\kappa t} - e^{\kappa t_1})\right]$$

Putting these two together, the derivative of the current mortality rate with reference to the age at which education terminates is

$$\begin{aligned}\frac{d\log s_t}{dt_1} &= \mu e^{-\mu t_1}\left[\rho(t-t_1) + \frac{\gamma}{\kappa}(e^{\kappa t} - e^{\kappa t_1})\right] \\ &= -\mu\left[\log s_t - \log s_{t_1}\right]\end{aligned}$$

or

$$\frac{ds_t}{dt_1} = -\mu s_t \log\frac{s_t}{s_{t_1}}$$

However, for $t < t_1$, in other words with education continuing, with $s_0 = 1$

$$\begin{aligned}\frac{d\log s_t}{dt} &= -(\rho + \gamma e^{\kappa t})e^{-\mu t} = -\rho e^{-\mu t} - \gamma e^{(\kappa-\mu)t} \\ \log\frac{s_t}{s_0} &= \frac{\rho}{\mu}[e^{-\mu t} - 1] - \frac{\gamma}{\kappa-\mu}[e^{(\kappa-\mu)t} - 1]\end{aligned}$$

Therefore, with $s_0 = 1$

$$\begin{aligned}\log s_{t_1} &= \frac{\rho}{\mu}[e^{-\mu t_1} - 1] - \frac{\gamma}{\kappa-\mu}[e^{(\kappa-\mu)t_1} - 1] \\ \frac{d\log s_{t_1}}{dt_1} &= -\rho e^{-\mu t_1} - \gamma e^{(\kappa-\mu)t_1}\end{aligned}$$

Life expectancy at age t is of course given as

$$\int_t^\infty s_t ds_t$$

but is usually evaluated numerically using mid-point summation. With the specification adopted we can see that it is a function of $e^{e^{-\mu t_1}}$.

3.3 Marginal Conditions with Endogenous Survival

The optimisation problem is just as before

$$
\begin{aligned}
MaxV &= \int_0^T s_t(t_1)e^{-\delta t}\left\{\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\tilde{c}^{1-\sigma}}{1-\sigma} - (\zeta_t + \chi_t)\,\phi(h_t)\right\}dt \\
&\quad +\lambda\int_{t_1}^{t_2} e^{gt}w_0 e^{\int_0^{t_1}\theta(\tau)d\tau} s_t\left(t_1\right)e^{-rt}dt - \lambda\int_0^T c_t s_t\left(t_1\right)e^{-rt}dt
\end{aligned}
$$

noting that $\chi_t = 0$ if $t < t_1$ and $ds_t/dt_1 = 0$ if $t < t_1$. However the fact that mortality rates are a function of years of education affects the derivatives with respect to t_1.

Differentiating with respect to t_1

$$
\begin{aligned}
\frac{dV}{dt_1} &= \int_{t_1}^T \frac{ds_t(t_1)}{dt_1}e^{-\delta t}\left\{\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\tilde{c}^{1-\sigma}}{1-\sigma} - (\zeta_t + \chi_t)\,\phi(h_t)\right\}dt \\
&\quad -\lambda e^{(g-r)t_1} s_t\left(t_1\right) w_0 e^{\int_0^{t_1}\theta(\tau)d\tau} \\
&\quad +\lambda\int_{t_1}^{t_2} e^{(g-r)t_1} w_0 e^{\int_0^{t_1}\theta(\tau)d\tau}\left(s_t\left(t_1\right)\theta(t_1) + \frac{ds_t(t_1)}{dt_1}\right)dt \\
&\quad -\lambda\int_{t_1}^T c_t e^{-rt}\frac{ds_t(t_1)}{dt_1}dt
\end{aligned}
\tag{5}
$$

If mortality is in fact insensitive to education, so that $ds_t/dt_1 = 0$, then the expression reduces to

$$
\frac{dV}{dt_1} = -\lambda e^{(g-r)t_1} s_t\left(t_1\right) w_0 e^{\int_0^{t_1}\theta(\tau)d\tau} + \lambda\int_{t_1}^{t_2} e^{(g-r)t_1} w_0 e^{\int_0^{t_1}\theta(\tau)d\tau} s_t\left(t_1\right)\theta(t_1)dt
$$

found in section 3.1. As noted there, when $\mu = 0$, this second expression is independent of both $\tilde{c}$, subsistence consumption, and λ, the marginal utility of life-time resources, while more generally the decision depends on these.

The sensitivity of mortality to education introduces three extra terms. The first, shown in the first row, shows the incremental benefit arising from the joy of living. We might follow Murphy & Topel (2006) and put a money value on the marginal increase in welfare arising through this route. The current increase in welfare arising from (certain) extra income of £1 at time t is λe^{-rt}. The current increase in discounted utility of unit increase in the probability of survival arising as a result of the joy of living is

$$
e^{-\delta t}\left\{\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\tilde{c}^{1-\sigma}}{1-\sigma} - (\zeta_t + \chi_t)\,\phi(h_t)\right\}
$$

so the value of a life year can be defined as

$$
\text{VLY}=\frac{e^{(r-\delta)t}}{\lambda}\left\{\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\tilde{c}^{1-\sigma}}{1-\sigma} - (\zeta_t + \chi_t)\,\phi(h_t)\right\} \tag{6}
$$

This shows the money value at time t of the discounted increase in life-time welfare per unit increase in survival at age t, and therefore can be interpreted as the value of a life year at age t. The expression shows that a life year is more valuable to someone who enjoys leisure than to someone at an age where it is optimal to allocate time to work or to study. More generally, it is possible, at least in principle, that life may not be worth living for someone who is working, but becomes worthwhile only after retirement. In particular, if the disutility of work falls steeply as age-dependent health status declines, then it is possible that utility may be negative shortly before retirement.

The second and third additional terms,

$$\lambda \int_{t_1}^{t_2} e^{(g-r)t_1} w_0 e^{\int_0^{t_1} \theta(\tau)d\tau} \frac{ds_t(t_1)}{dt_1} dt - \lambda \int_{t_1}^{T} c_t e^{-rt} \frac{ds_t(t_1)}{dt_1} dt$$

represent the marginal impact on utility of the financial consequence of changed mortality. Extra resources are available because of the increased chance of survival during working life, but these are offset by the increased expected cost of higher survival rates. One might expect that the absolute impact on survival rates of any effect from education will tend to come late in life and that therefore, discounting notwithstanding, the overall financial effects of higher survival rates will tend to be to depress utility. These affect the finances of the marginal decision around any particular level of of education and need to be taken into account in addition to the usual calculation balancing up the cost of incremental education against the income benefits when survival is exogenous.

We can now set out the three first-order conditions with our parametric specification of the link between education and survival rates. The first-order condition for the age of completion of education, t_1 becomes

$$\begin{aligned} \frac{dV}{dt_1} &= -\int_{t_1}^{T} \mu s_t \log \frac{s_t}{s_{t_1}} e^{-\delta t} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{\tilde{c}^{1-\sigma}}{1-\sigma} - (\zeta_t + \chi_t)\,\phi(h_t) \right\} dt \\ &\quad -\lambda e^{(g-r)t_1} s_t(t_1)\, w_0 e^{\int_0^{t_1} \theta(\tau)d\tau} \\ &\quad +\lambda \int_{t_1}^{t_2} e^{(g-r)t_1} w_0 e^{\int_0^{t_1} \theta(\tau)d\tau} \left(s_t(t_1)\,\theta(t_1) - \mu s_t \log \frac{s_t}{s_{t_1}} \right) dt \\ &\quad +\lambda \int_{t_1}^{T} c_t e^{-rt} \mu s_t \log \frac{s_t}{s_{t_1}} dt = 0 \end{aligned} \tag{7}$$

The first-order condition for consumption remains unchanged. With the rate of discount equal to the rate of interest, consumption is constant,

$$c_t = c^* \text{ and } \lambda = (c^*)^{-\sigma} \tag{8}$$

The age of retirement is given, as before, by the point at which the disutility of work just equals its marginal benefit. Both of these are conditional on expected survival, but the marginal decision does not depend on the probability of survival.

$$\phi(h_t) = \lambda e^{gt} w_0 e^{\int_0^{t_1} \theta(\tau)d\tau} \tag{9}$$

Since these last two conditions are independent of survival, the only marginal decision affected by a connection between education and survival is the marginal decision to undertake education. The outcomes for consumption and retirement will nevertheless be affected because of the consequences of educational attainment for survival, in much the same way as they would be influenced by an exogenous change to survival rates.

4 A Practical Difficulty

Many models of the effect of education assume that there is a log-linear influence of education on subsequent earnings power. The original study by Mincer (1974) estimated a return of around seven per cent to each year of education. More recent studies suggest that the return declines with the total amount of education. In this section I show that the simple assumption of a linear model yields implausible results. The problem can be seen very clearly by looking at a simple model with a fixed retirement age, $t_2 = 65$, and no mortality risk. . In that case the problem faced by the individual is to maximise the discounted value of labour income between age t_1 at which education finishes and t_2. With r the interest rate and g the growth rate, the problem is simply

$$\underset{t_1}{Max} \; e^{\theta t_1} \int_{t_1}^{t_2} e^{(g-r)t} dt$$

The first-order condition for this is

$$\frac{\theta e^{\theta t_1}}{(r-g)} \left(e^{(g-r)t_1} - e^{(g-r)t_2} \right) - e^{(g-r)t_1} e^{\theta t_1} = 0$$

and the value of t_1 which satisfies this is easily found numerically. Table 4 shows the values of t_1 which result from different combinations of r and θ with $g = 0.012$. A plausible value of t_1 is obtained with $\theta = r = 0.04$, but for the most part the optimisation points to an age for the completion of education substantially higher than is generally observed. For values of θ of 0.05 or higher it is possible to obtain ages at which education ends in the region of twenty or so, only if the interest rate is around 0.005 or less below the value for the return to education. With the linear model it is not possible, then, to reconcile observed periods of study with observed interest rates and observed returns to education. This difficulty is eased only slightly by taking account of the risk of death before retirement, because that risk is not very great. A man who reached the age of sixteen in 1987 is estimated to have a ninety per cent chance of surviving from sixteen to sixty-five according to the historic and projected mortality rates published by the UK Office for National Statistics.

	θ									
r	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%
1%	40.6	45.4	48.6	50.9	52.7	54.0	55.1	56.0	56.7	57.4
2%	37.1	43.2	47.1	49.8	51.8	53.4	54.6	55.6	56.4	57.1
3%	31.8	40.2	45.2	48.5	50.8	52.6	54.0	55.1	56.0	56.7
4%	22.0	35.7	42.5	46.8	49.6	51.7	53.3	54.5	55.5	56.3
5%		27.4	38.6	44.4	48.0	50.6	52.4	53.8	55.0	55.9
6%			31.5	40.9	45.9	49.1	51.4	53.1	54.4	55.4
7%			6.4	34.6	42.7	47.2	50.0	52.1	53.6	54.8
8%				12.7	37.1	44.3	48.2	50.8	52.7	54.1
9%					17.7	39.2	45.6	49.2	51.5	53.3
10%						21.7	40.9	46.7	50.0	52.2

Table 1: The Optimal Age to End Education with Constant Returns to Education

There are two possible ways in which this difficulty could be resolved. One is to produce a model which takes account of the fact that the return to education for any individual is uncertain. This means that a risk-averse individual will treat the return to education as lower than it actually is, when deciding how much education to undertake. An alternative, and much simpler framework is provided by the assumption that the rate of return to education is a diminishing function of the amount of education undertaken. For example

$$w = e^{\theta_1(t_1-a)^2+\theta_2(t_1-a)}.$$

In this case

$$\frac{d\log w}{dt_1} = \theta(t_1) = 2\theta_1(t_1-a)+\theta_2$$

which is diminishing in t_1 if $\theta_1 < 0$.

This model implies that there is an age

$$t_{1\max} = a - \frac{\theta_2}{2\theta_1};$$

beyond which the marginal return to education is negative. It is worth noting that Rudd (1990) famously reported negative returns to PhDs while Weale (1992) identified very poor returns to some types of degrees, at least as measured by earnings seven years after graduating.

The first-order condition becomes, if the retirement date is fixed at t_2 and in the absence of any influence on survival rates,

$$\frac{(2\theta_1(t_1-a)+\theta_2)^{\theta_1(t_1-a)^2+\theta_2(t_1-a)}}{(r-g)}\left(e^{(g-r)t_1}-e^{(g-r)t_2}\right)-e^{\theta_1(t_1-a)^2+\theta_2(t_1-a)}e^{(g-r)t_1}=0$$

Table 2 shows the age at which education is ended for different values of θ_2 and r with $g = 0.012$ as before, $\theta_1 = -0.002$ and $a = 0$ implying that the maximum benefit from education is obtained with an age of 27.5 years. This shows much more clustering around the ages at which full-time education normally finishes. Table 3 shows the marginal log return to education, measured as $2\theta_1(t_1 - a) + \theta_2$ for each of the terminal ages indicated in table 2. It can be seen that plausible values of the terminal age of education are achieved by means of fairly rapidly diminishing marginal benefits. These stand somewhat at odds with some of the reported returns to degree-level qualifications, but it must be remembered that these are measures of the return to people who actually gain their degrees and do not necessarily indicate the average benefit of study at that level.

$\theta_1 = -0.002, a = 0$	θ_2									
r	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%
1%	15.2	17.5	19.7	21.9	24.1	26.3	28.4	30.5	32.5	34.5
2%	14.0	16.3	18.6	20.8	23.0	25.2	27.3	29.4	31.5	33.5
3%	12.6	14.9	17.2	19.5	21.7	23.9	26.1	28.2	30.3	32.4
4%	11.0	13.3	15.7	17.9	20.2	22.4	24.7	26.8	29.0	31.1
5%	9.2	11.6	13.9	16.2	18.5	20.8	23.1	25.3	27.5	29.6
6%	7.2	9.6	12.0	14.3	16.7	19.0	21.3	23.6	25.8	28.0
7%	5.0	7.5	9.9	12.3	14.7	17.0	19.4	21.7	24.0	26.3
8%		5.2	7.6	10.1	12.5	14.9	17.3	19.7	22.0	24.4
9%			5.3	7.8	10.2	12.7	15.1	17.5	19.9	22.3
10%				5.4	7.9	10.3	12.8	15.2	17.7	20.1

Table 2: The Optimal Age at which to Complete Education with Diminishing Returns to Education

The simple life-cycle framework now generates a duration for education similar to the sorts of figures that we see in practice, but with marginal returns still somewhat lower than the benefits associated with incremental years of education.

$\theta_1 = -0.002, a = 0$	θ_2									
	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%
1%	1.9%	2.0%	2.1%	2.2%	2.4%	2.5%	2.6%	2.8%	3.0%	3.2%
2%	2.4%	2.5%	2.6%	2.7%	2.8%	2.9%	3.1%	3.2%	3.4%	3.6%
3%	3.0%	3.0%	3.1%	3.2%	3.3%	3.4%	3.6%	3.7%	3.9%	4.1%
4%	3.6%	3.7%	3.7%	3.8%	3.9%	4.0%	4.1%	4.3%	4.4%	4.6%
5%	4.3%	4.4%	4.4%	4.5%	4.6%	4.7%	4.8%	4.9%	5.0%	5.1%
6%	5.1%	5.2%	5.2%	5.3%	5.3%	5.4%	5.5%	5.6%	5.7%	5.8%
7%	6.0%	6.0%	6.1%	6.1%	6.1%	6.2%	6.2%	6.3%	6.4%	6.5%
8%		6.9%	6.9%	7.0%	7.0%	7.0%	7.1%	7.1%	7.2%	7.3%
9%			7.9%	7.9%	7.9%	7.9%	8.0%	8.0%	8.0%	8.1%
10%				8.9%	8.9%	8.9%	8.9%	8.9%	8.9%	9.0%

Table 3: The Marginal Return to Education at the Optimal Age

5 Modelling Mortality Risk

Our model is applied to men because they have a clearer pattern of labour market participation and retirement than do women, particularly if one looks at the past as well as the present. We look first at men born in 1906. The *Human Mortality Database*[4] provides life tables for the United Kingdom from 1922 to 2011, and thus covers this cohort from the age of sixteen until the age of one hundred and five, by which time there are very few survivors. Men in this age group were in active service during the Second World War. The effects of the war on mortality rates are removed by interpolating between the mortality rates for 1939 and 1946. Obviously it is possible that subsequent mortality rates were affected by war service; there is no obvious way of addressing this. The second set of mortality rates are those published by the Office for National Statistics. These show actual or forecast mortality rates for men of different ages in each year from 1981 to 2062. Since we focus on those aged sixteen or above, we can consider men born between 1981 and 2062. Figure 2 shows log mortality rates for 1906 cohort indicating both the raw data and the data with the effects of the War removed. Secondly, it shows log mortality rates for the cohort born in 1981, with the path for the latter beyond age 55 being a forecast. Finally, it shows the projections for the men to be born in 2062. The mortality rates for these are closely modelled on the pattern for the 1981 cohort. These differences in mortality rates have substantial implications for life expectancy.

In section 3.2.1 we described the functional form used to explain mortality rates,

[4] The *Human Mortality Database,* http://www.mortality.org/, appears to provide mortality rates from the age of sixteen for men born in 1890. There are, however, doubts about the data presented.

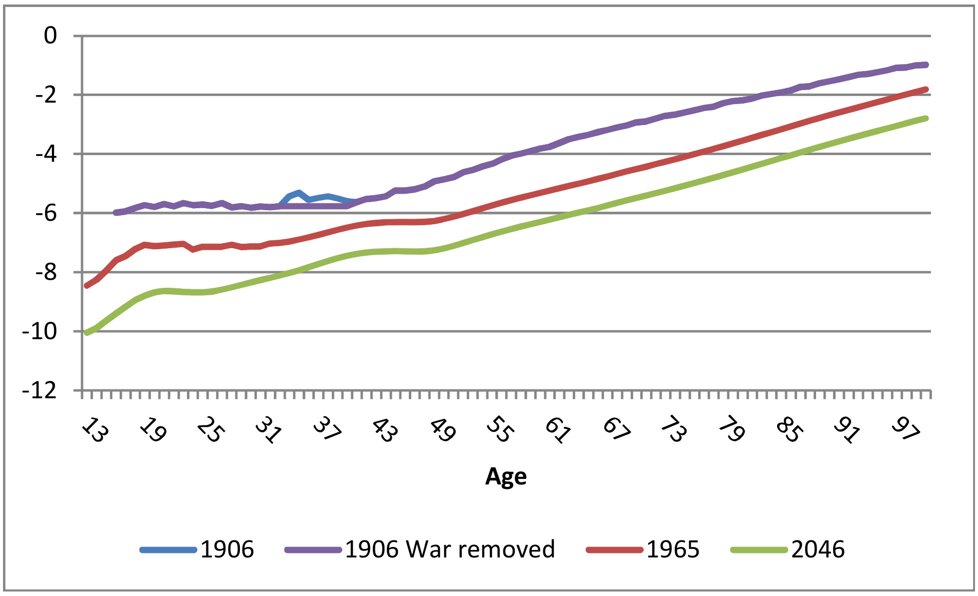

Figure 2: Log Mortality Rates for Men born in the United Kingdom in 1906, 1965 and 2046

estimated as

$$\log m_t = \log(\rho + \gamma e^{\kappa t}) + \eta_t$$

Here η_t is a residual term, linking the published mortality rates to those generated by the functional form.

Figure 3 shows the estimated parameters for the cohorts of men born from 1981 to 2062. Actual mortality rates are observed up to 2012 and these are reflected in the calculations for the early cohorts. Rising longevity is assumed to arise from i) a reduction in the component of mortality which is independent of age, ρ, and ii) a reduction in the impact of the age-dependent effect, γ. The rate of growth of the age-dependent effect, κ, is, however, assumed to change very little over time. For the 1906 cohort a rather different picture is obtained with, $\rho = 0.002, \gamma = 9.034 \times 10^{-5}$ and $\kappa = 0.0868$. In other words the rate of age-independent mortality is much higher, reflecting, for example. the risks resulting historically from infectious diseases.

One might hope that, given information on the age at which each cohort, or at least the early cohorts terminated their education, it might also be possible to estimate μ. Since extension of education has proceeded in step with innumerable other changes that have had the effect of raising life expectancy, that is not really possible. Instead, therefore, we specify a value for μ and estimate the other parameters conditional on that. It should be noted that our model of the effects of education, looks at the age at

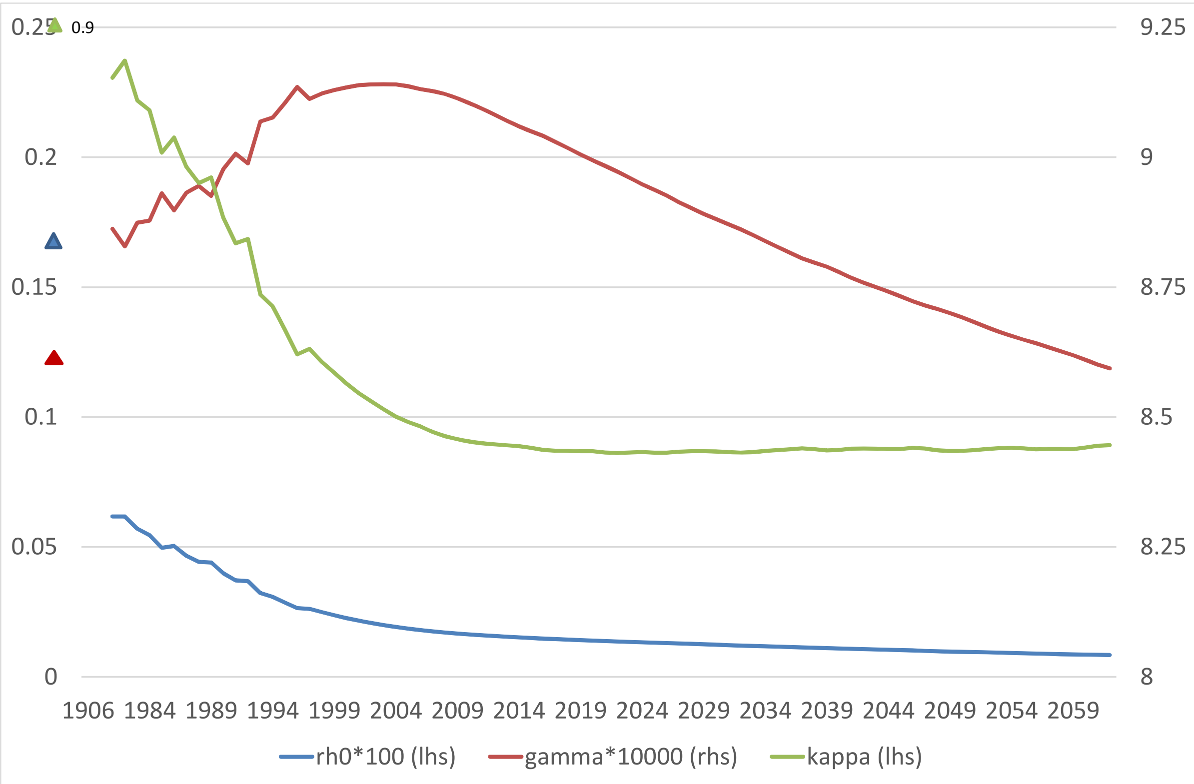

Figure 3: The Parameters of the Mortality Model (1906 figures shown as triangles)

which education was completed rather than the time spent in education. This has no impact on our findings because first the age at which compulsory education in the United Kingdom started has remained at five throughout the period. Thus rebasing to years spent in education would have the effect of multiplying the mortality rate by a constant term. Since, however, the parameters, ρ and γ are estimated freely, the replacement of $e^{-\mu t}$ by $e^{-\mu(t-t_b)}$, where t_b is the base age from which the effect starts to apply, has no material impact the subsequent calculations. Equally, the structure of the model is such that the other parameters adjust fully to offset the effects of any selected value of μ, so the choice of μ has no bearing on the ability of the model to fit the mortality data of any particular cohort.

The parameter μ was set to 0.05. Figure 4 shows the effect on the probability of surviving to a given age on the cohort of men born in 1981 of increasing the age at which education was completed from seventeen to twenty-one. Figure 5 shows the consequence of this for life expectancy as a function of age. At the age of sixty-five, four years of education raises life expectancy by 1.7 years. Notwithstanding the debate, summarised in the introduction about the evidence for a link between education, or factors related to education such as income, and survival, this effect is probably rather less powerful than suggested by those who do find evidence for a link. As Figure 4 makes clear, and not very surprisingly, the main absolute effect on survival rates comes late in life. An equiproportional reduction in mortality has much more absolute impact when mortality rates are high than when they are low, and thus the effect is largely felt in old age. As a consequence of that, discounting will sharply attenuate the benefits seen from the perspective of a young person making decisions about investment in human capital .

6 Simulation of the Model

6.1 Exogenous Mortality

In addition to the mortality functions described above. the model has a number of parameters which have to be set in order to explore the relationship between education, retirement and mortality. It is necessary to specify the relationship between the disutility of education/work and health, h_t Following Bloom et al. (2014), we set the disutility of work as being proportional to the mortality rate, $\phi(h_t) = \phi m_t$. We set the parameters determining the effects of education, the wage growth rate and the discount rate at fixed values shown in table 4. . The value chosen for the rate of return is an estimate of the

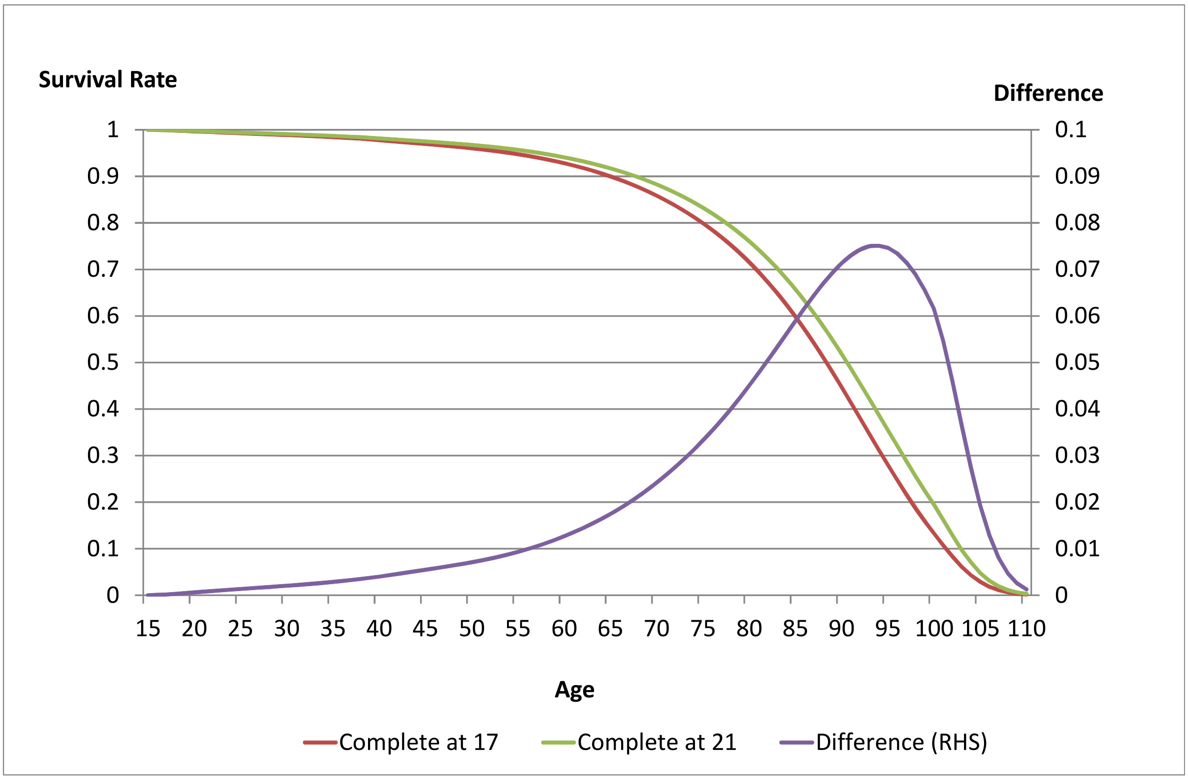

Figure 4: The Effects of Four Extra Years of Education on Survival ($\mu = 0.05$)

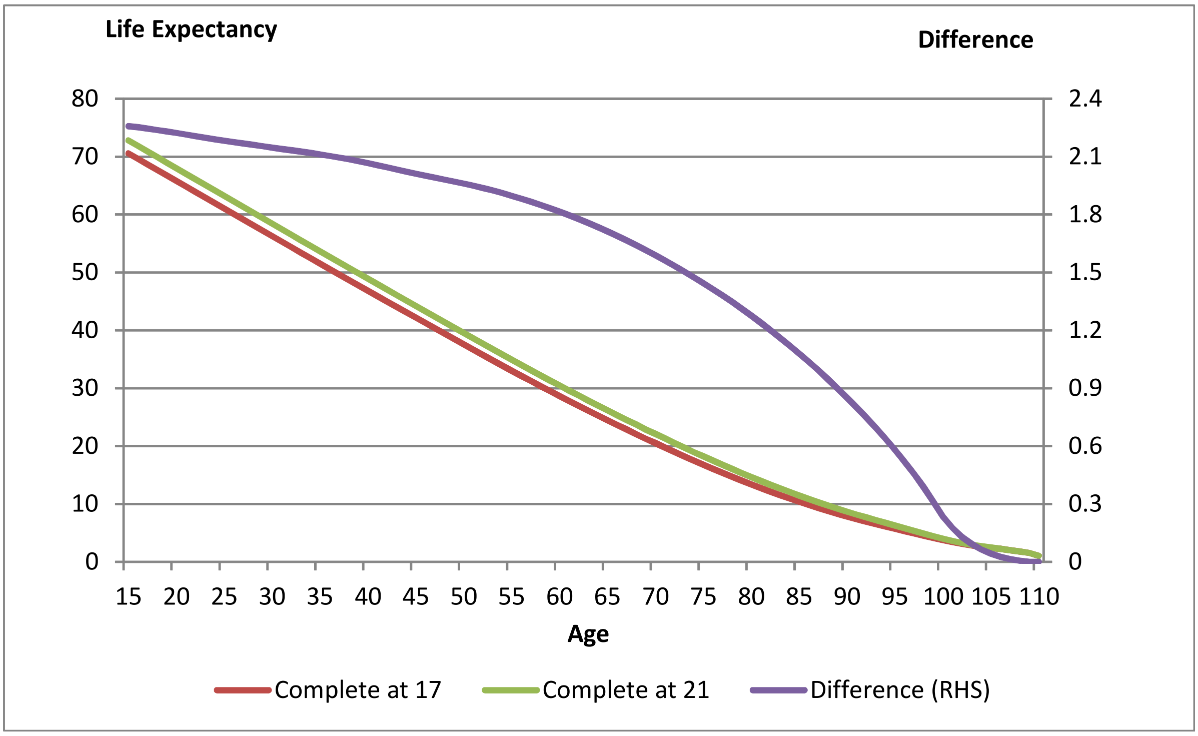

Figure 5: The Effects of Four Extra Years of Education on Survival Rates ($\mu = 0.05$)

Variable		**Value**
Interest/Discount Rate	r, δ	0.04
Growth of Wage Rate	g	0.012
Return to Education: Quadratic Term	θ_1	-0.002
Return to Education: Linear Term	θ_2	0.11
Age age at which education gives max wage	$t_{1\max}$	27.5
Reciprocal of Intertemporal Elasticity of Substitution	σ	2
Wage for someone educated to sixteen	w_0	See text
Disutility of work	ϕ	85

Table 4: Model Parameters

return on private capital (including land and housing) in the period before the crisis. This was found to be of the order of 4% p.a. or slightly higher[5] in both France and the United Kingdom. The rate of growth of wages is set well below the rate of growth of overall labour productivity before the crisis. This reflects the fact that the latter was a consequence of cohort effects, themselves in part due to improving educational attainment, as well as "pure" technical progress; historically each cohort's earnings have grown more slowly than the real wage rate in the economy as a whole.

The reciprocal of the inter-temporal elasticity of substitution of 2 is higher than is often suggested and used by macro-economists (Lucas 1990). It is, however, at the low end of the range suggested by life-cycle models of household behaviour (Hall 1988, Sefton et al. 2008). Finally, for any given cohort, the interplay of the education and retirement decisions depends on w_0, the wage which multiplies up the education effect. In the absence of any link between education and survival rates, the assumed subsistence level of education does not affect the result.

A feature of the model is that the outcome depends on relative wage rates. For the 1906 and 1981 cohorts we compare the real hourly wages when the men in our cohorts were aged around forty. For those born in 1906 it makes more sense to look at average hourly pay in 1948 than immediately after the end of the Second World War. This, in real terms of 29.5 per cent of the real wage in 2006. However, a part of this difference is due to education.

Before 1920 education was compulsory only up to the age of thirteen and not many children stayed on beyond this. it is reasonable to assume that the average age for completing education was fourteen. The school-leaving age was set at sixteen in 1972 and the majority of children continued to leave at the minimum age until the late 1980s.

[5]This rate reflects the combination of apprecialy higher returns in the corporate sector and low returns on land and housing.

The British Cohort Survey, a study of children born in 1970, shows that for boys the average age of completing education was 18.1 years; it is unlikely this is very different from the figure for boys born in 1981. The earliest year for which data on average retirement age are available is 1984; this shows an age of 63.7 for men; it is, however, likely that retirement ages fell during the 1970s and particularly during the recession of 1979-1981. The retirement age for men born in 1981 is, of course, unknown. Their state pension age will, however, be sixty-seven.

With the selected parameters our model of the effects of education implies that real pay would have grown by 12% as a result of the increased education. That means that, on a like for like basis, we set the real earnings of those born in 1906 at 0.33 compared to 1 for those born in 1981. We assume that the underlying increase in productivity takes this to 1.5 for those to be born in 2062.

The results of this are shown in table 4. Between 1906 and 1981 there has been a sharp increase in life expectancy together with a large increase in wages and incomes. These two influences have opposing effects on the retirement age so that, with the parameters adopted, it is only slightly higher for the men born in 1981 compared to those born in 1906. While the wage (net of the effects of education) grew three-fold between 1906 and 1981. it is assumed only to rise by about fifty per cent for the men born in 2062, while life expectancy rises by a further nine years. The weaker income effect means that the retirement age should rise into the mid seventies, at least with these model parameters.

It is striking, however, that while the effects discussed by Ben-Porath (1967) can be observed, they are not very powerful. Between the 1906 and 1981 cohorts, the mortality rates for men of working age fall to fairly low levels, making education more worthwhile, as Cervellati & Sunde (2013) discuss. But, partly because the interest rate/discount rate is assumed to be appreciably above the growth rate, the discounted impact of this on life-time earnings is fairly small and the effect on education is therefore not large. The same point arises for the 2062 cohort. The main reduction in mortality now comes well after the end of working life, and this is highly discounted. Of course these results are shown only for one particular parameter set; it is quite straightforward to adjust the parameters so that the retirement ages move up or down and closer or further apart. It does not seem possible, however, despite searching over a wide range of parameters, to come up with an increase in the age of completing education of anything like the four years or so observed in the data.

Table 6 makes it possible to understand the importance of rising wages as a factor

	1906	1981	2062
Life Expectancy Age 21	49.6	65.6	77.4
Life Expectancy Age 50	25.0	38.3	49.1
w_0	0.33	1	1.5
Annual Consumption	1.48	5.00	7.82
Age Complete Education	17.2	18.1	18.7
Marginal Rate of Return	4.1%	3.8%	3.5%
Retire	64.5	69.1	76.2

Table 5: Simulation Results with Exogenous Mortality

	1906	1981	2062
Life Expectancy Age 21	49.6	65.6	77.4
Life Expectancy Age 50	25.0	38.3	49.1
w_0	1	1	1
Annual Consumption	4.30	4.99	5.31
Age Complete Education	15.9	18.1	18.6
Marginal Rate of Return	4.8%	3.6%	3.5%
Retire	51.9	69.1	80.4

Table 6: Results with Constant Underlying Wages

behind declining retirement, or at least as an explanation of why retirement ages have changed little despite rising longevity. The underlying wage rate is assumed not to change over time in the simulations leading the results shown here. The retirement age falls sharply for the oldest cohort. given their mortality pattern, the higher income means that they choose to take more leisure in old (or not so old) age. Earlier retirement means that education is less worthwhile, so the age of completing education drops. For the men born in 2062 the opposite happens. They work longer because lower wage rates mean that leisure is less affordable. Working longer makes education seem more worthwhile. and rises for the youngest. These changes have knock-on effects on the age of completing education. Since the first cohort retires earlier, education is less valuable, and is completed sooner. For the oldest cohort, by contrast, it is more valuable and is completed later than when the base wage was higher. This illustrates a very plausible point. When a high wage can be obtained without education, there is less incentive to meet the costs of obtaining education. This is an example of Le Chatelier's principle (Samuleson 1960).

6.2 Comparison with Empirical Findings

It is worth asking how far these results might relate to the relationship found in section 2. Of course the results there were in terms of participation in education from age fifteen to twenty-four rather than the age at which education was completed. We can, however, make some comparisons. Comparing the 1981 and 2062 cohorts, modelled life expectancy at age fifty has increased from 38.3 to 49.1 years. If the initial participation rate were 0.5, then the increase in life expectancy implied by the empirical model would be associated with an increase in the participation rate to 0.81. If, furthermore, one makes the assumption that the age of completing education is equal to 15 plus 9×participation rate, this implies an increase in age of completion of 2.7 years. While the figures of table 6.3 show lower ages of completing education, they also suggest that the observed effect of section 2 is something like three times as powerful the simulated effect.

Applying the same approach to the increase in life expectancy at age fifty between the 1906 cohort and the 1981 cohort results in an increase in years of education of 3.9. This is very different from the figure of 0.9 years which is simulated, but close to the observed figure of four years. Thus UK experience between 1906 and 1981 is coherent with what is observed for the EU over a much shorter period, but is much more powerful than can be generated by a straightforward life-cycle model. The comparison has to be seen as very approximate. If the initial participation rate were much lower or much higher than the assumed 0.5, then the implied impact of rising life expectancy would be lower. Nevertheless, it would have to be very low or very high for the effect of actual changes in life expectancy at age fifty to be as low as those implied by table 6.3.

6.3 Endogenous Mortality

When mortality is endogenous, the effect of education leads to a number of effects pulling in different directions. The level of subsistence consumption relative to actual consumption, plays an important role. As equation (7) shows, the marginal decision is subject to a number of different influences. First of all, provided that the present discounted value of utility is in fact positive, so that overall life is worth living, then a reduction in the mortality rate will have a favourable influence on this. Secondly, it makes the expected return from working higher because it reduces the risk of death during working life. Thirdly, however, it means that any given amount of discounted life-time consumption has to be spread more thinly. This brings down the flow of consumption,

	1906	**1981**	**2062**
Life Expectancy Age 21	51.8	66.6	77.7
Life Expectancy Age 50	26.7	39.2	49.3
w_0	0.33	1	1.5
Subsistence Consumption	1.2	1.2	1.2
Annual Consumption	1.61	5.37	8.28
Age Complete Education	17.5	19.7	20.4
Marginal Rate of Return	4.0%	3.1%	2.9%
Retire	66.7	70.7	77.1

Table 7: Simulation Results with Endogenous Mortality

reducing welfare. This effect is likely more than to offset the positive impact on life-time labour income, because mortality rates are in any case relatively low during working life. Furthermore, if consumption is not much above subsistence, then the gain from the first component will be small. It is possible therefore that, even if life is worth living, a marginal reduction in mortality will not be worth having. With consumption well above subsistence, the benefit from increased marginal utility will become larger and go further to offset the depressing effect from spreading consumption more thinly. On top of this, the effect of extra education, undertaken mainly because of its impact on survival will have the consequence that retirement will be delayed somewhat; the higher wages resulting make working more worthwhile.

Using the same parameters as before, but with $\mu = 0.05$ rather than zero and with subsistence consumption, which did not influence the earlier results, set to 1.2, we find the results shown in table 6.3. These show that all three cohorts are educated for longer and retire later. The fact that increased longevity is worth more to a man born in 1981 than to someone born in 1906 means that the effect much more pronounced for this cohort. As a result a gap of over two years in completion of education is found, compared with 0.9 years in the absence of a mortality effect, and four years observed in practice. Equally striking is the fact that the impact of reduced mortality on the education of the 2062 cohort is much smaller. The gap between them and the 1981 cohort increases only from 0.6 years to 0.7 years. The reason for this is that, except in extreme old age, the mortality rate of this cohort is very low anyway. The benefits which accrue in extreme old age are heavily discounted and do not have much impact on people's choices.

If, however, the results are to be aligned to the experience of the cohort born in 1981, then it is necessary to reduces the marginal returns to education, resulting in all three

	1906	**1981**	**2062**
Life Expectancy Age 21	50.5	65.6	76.7
Life Expectancy Age 50	25.8	38.3	48.4
w_0	0.33	1	1.5
Subsistence Consumption	1.2	1.2	1.2
Annual Consumption	1.61	5.37	8.28
Age Complete Education	15.6	18.1	18.7
Marginal Rate of Return	4.0%	3.0%	2.7%
Retire	65.2	69.6	76

Table 8: Simulation Results with Endogenous Mortality

cohorts completing their education sooner. If the age at which education delivers its maximum benefit is reduced from 27.5 years to 26 years, the results shown in table 6.3 are obtained. It is now possible to account for two and a half years of the four-year gap in completing education between the 1906 cohort and the 1981 cohort. The retirement ages are also slightly earlier which is rather more plausible. It should be noted that, despite education being completed earlier, the consumption level is higher than when the maximum benefit of education is obtained at 27.5. This is almost entirely a consequence of the fact that the downward adjustment to the age of maximum benefit has the effect of raising the average return to education at the ages material to us, while nevertheless reducing the marginal return.

7 Conclusions

The education decision can be assessed in the context of the life-cycle model as an extension to the model developed by Bloom et al. (2014). The conventional assumption that Mincerian returns to education are of the order of seven per cent or so does not sit happily in a world where the return on capital is of the order of four per cent. It implies that education would continue until well into middle age. This difficulty does not arise if it is assumed that the returns to education decline with the amount of education received, and it becomes possible to simulate ages of completion of education of between fifteen and twenty, when the model is applied to the cohort mortality rates for men in the United Kingdom.

The model shows the sensitivity of education to mortality rates anticipated by Ben-Porath (1967). The empirical magnitudes of the effects are, however, small. The computed increase in years of education between the cohort of men born in 1906 and that born in 1981 is only 0.9 years, as compared with an actual increase of four years.

There is some, if by no means overwhelming, evidence that mortality rates are declining in educational attainment. The model can be extended to reflect this effect and, with an effect somewhat smaller than some researchers have found, the gap in years of education between the 1906 and 1981 cohorts does increase from under a year to $2\frac{1}{2}$ years. This result is, however, dependent on the presence of a subsistence level of consumption which amounts to around 3/4 of the simulated consumption of the 1906 cohort, but is relatively of much less importance to the 1981 and 2062 cohorts. A consequence of the subsistence consumption is that the marginal benefit of education, arising through longevity, is much smaller for the 1906 cohort than for the other cohorts.

To the extent that this is the mechanism, however, it carries the implication that years of education will not rise much further cohorts. This is a consequence of further declines in mortality rates, with the implication that the benefits of lower mortality rates flowing from education are pushed further into the future and therefore discounted more heavily. While it is, of course, much too early to say that such a change will not happen, recent trends in years of education in the United Kingdom have been in the opposite direction, with increasing participation in higher education, and an increase in the school leaving age to eighteen in 2015.

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